

Lesson-23

Cost Analysis-II

Explicit Costs and Implicit Costs

Economists have classified types of costs as explicit (accounting) costs, and implicit costs. Explicit costs are out of pocket, obvious kinds of costs, e.g., expenses on books, tuition, as, etc. Implicit costs are not really expenses you incur, but involve income or values you are giving up by not doing something that you could have chosen to do. For instance, if you decide to go to school full time instead of working a \$20,000 job, you are giving up earning \$20,000. This is your implicit cost.

Sample problem: Suppose you are running a small business and incur the following expenses: labor = \$80,000; raw materials = 30,000; finance charges on a loan = \$3,000. You are not paying explicit rent, because you own the building you are operating in. If you would rent it out, however, you could be earning \$12,000. You also estimate your own time to be worth \$25,000. What are your expenses?

Answer: **Explicit costs** = \$80,000 + \$30,000 + \$3,000 = \$113,000.
 Implicit costs = \$12,000 + \$25,000 = \$37,000.
 Total economic costs = explicit + implicit costs = \$150,000

Accounting vs. Economic Costs

Accountants have been primarily concerned with measuring costs for financial reporting purposes. As a result, they define and measure cost by the historical outlay of funds that takes place in the exchange or transformation of a resource.

Economists have been mainly concerned with measuring costs for decision-making purposes. The objective is to determine the present and future costs (or resources) associated with various alternative courses of action.

In calculating the cost to the firm of producing a given quantity of output, economists include some additional costs that are typically not reflected in financial reports.

- Explicit cost are considered by both groups
- Implicit costs are considered by economists:
 - a. Opportunity cost of time
 - b. Opportunity cost of capital

Economic Profit = Tot Rev - Exp Cost - Imp Cost

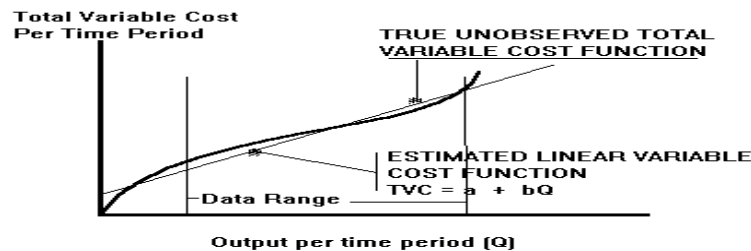
Accounting versus Economic Profits

To calculate accounting and economic profits we need to know the company's total revenue. Let's suppose it is \$140,000. Then, accounting profits are: total revenue minus explicit costs or: $\$140,000 - \$113,000 = \$27,000$. Economic profits are: total revenue minus total economic costs or: $\$140,000 - \$150,000 = -\$10,000$ (i.e. a loss of \$10,000). The above firm reaps a positive accounting profit; but the negative economic profit indicates that from an economic point of view, the owner should discontinue the operation.

Total and Per Unit Costs

Seven cost functions, which we will discuss, are:

1. Total Variable Cost (TVC) – The cost of all variable resources.
Examples: Cost of labor, materials, office supplies.
2. Total Fixed Cost (TFC) – The cost of all fixed inputs. Examples: Cost of the building, large pieces of machinery, certain taxes
3. Total Cost (TC) – This is the sum of TVC and TFC.
4. Average Variable Cost (AVC) – This is variable cost per product.
5. Average Fixed Cost (AFC) – This is fixed cost per product.
6. Average Total Cost (ATC) – This is total cost per product.
7. Marginal Cost (MC) – This is the cost of producing an additional unit of the product



Cost Calculations

Using the above abbreviations and Q for the quantity of output:

$$ATC = TC/Q$$

$$AFC = TFC/Q$$

$$AVC = TVC/Q$$

$$MC = \text{change in } TC / \text{change in } Q$$

Example: Let's suppose you are making 50 bottles of wine each week. You know that your fixed costs add up to \$300, and your variable costs amount to \$900. You also know that if you were to make extra 5 bottles, your total cost would rise by \$60. What is your total cost; average total cost; average total cost; average variable cost; average fixed cost; and marginal cost?

Answer: Total cost = \$300 + \$900 = \$1200

ATC = \$1200/50 = \$24

AVC = \$900/50 = \$18

AFC = \$300/50 = \$6

MC = \$60/5 = \$12

A table with cost data might show the following:

Q	TC	TFC	TVC	ATC	AFC	AVC	MC
45	1150	300	850	25.55	6.66	18.88	
50	1200	300	900	24.00	6.00	18.00	10
55	1260	300	960	22.91	5.45	17.45	12
60	1380	300	1080	23.00	5.00	18.00	24

- **Ore Mining Example-- Cost Data**

Q	L	VC	FC	TC	AFC	AVC	ATC	MC
0	0	0	150	150	150	0	150	
6	1	50	150	200	25.00	8.33	33.33	8.33
16	2	100	150	250	9.38	6.25	15.63	5.00
29	3	150	150	300	5.17	5.17	10.34	3.85
44	4	200	150	350	3.41	4.55	7.95	3.33
55	5	250	150	400	2.73	4.55	7.27	4.55
60	6	300	150	450	2.50	5.00	7.50	10.00
62	7	350	150	500	2.42	5.65	8.06	25.00
62	8	400	150	550	2.42	6.45	8.87	ERR
61	9	450	150	600	2.46	7.38	9.84	-50.00
59	10	500	150	650	2.54	8.47	11.02	-25.00

- Rent of the ore-mining equipment = \$0.20 per horsepower (750 x 0.20 = \$150)
- Cost of each worker employed is \$50 per period.

1. $TC = FC + VC$

2. $AFC = \frac{FC}{Q}$

3. $AVC = \frac{VC}{Q}$

4. $ATC = \frac{TC}{Q}$ or $AFC + AVC$

5. $MC = \frac{\Delta TVC}{\Delta Q}$, note the following
 $TVC = a + bQ$, where

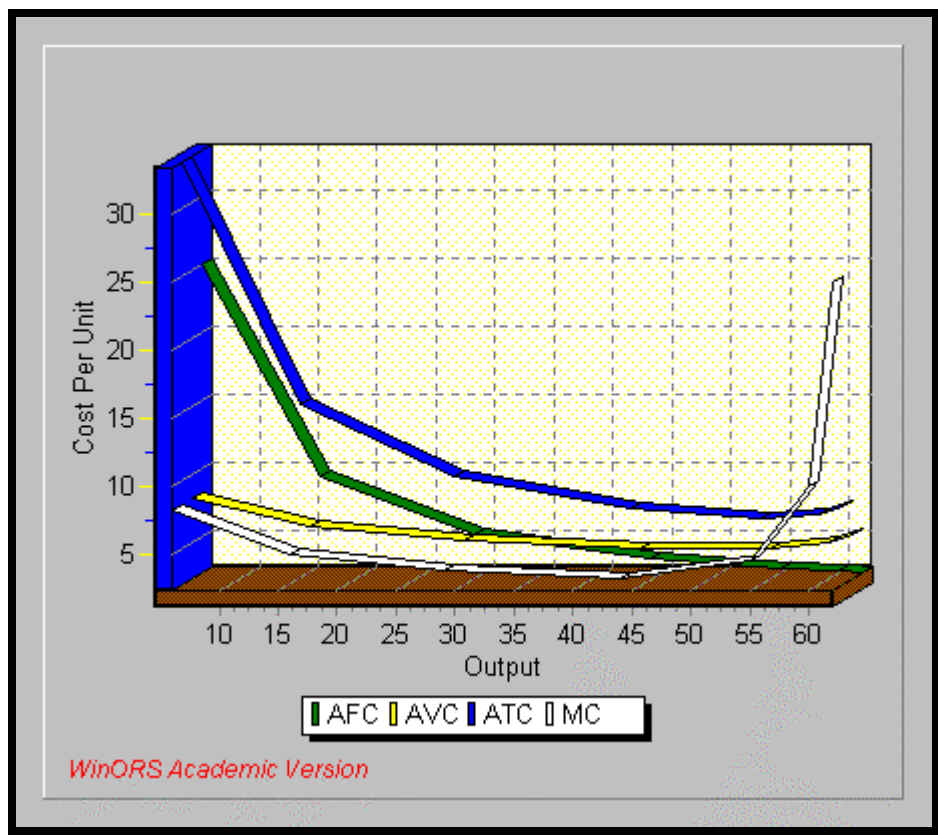
$$b = \frac{\Delta TVC}{\Delta Q} = MC$$

6. Cost Elasticity $\varepsilon = \frac{\% \Delta TC}{\% \Delta Q} = \frac{\Delta TC}{\Delta Q} \cdot \frac{Q}{TC}$

$\varepsilon < 1$ increasing returns to scale

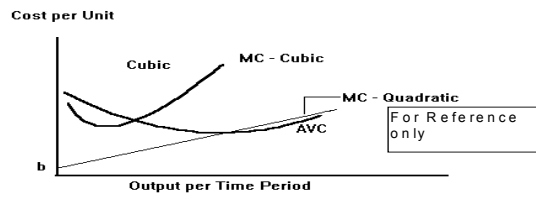
$\varepsilon = 1$ constant

$\varepsilon > 1$ decreasing



1. Cubic Form (with Quadratic MC):

Cubic-- Assumes that both marginal cost and average variable cost functions have U-shapes.



$$VC = a + bQ - cQ^2 + dQ^3 \quad a > 0, b < 0, \text{ and } c > 0$$

Thus, MC is U-shaped

$$MC = b - 2cQ + 3dQ^2$$

Example:

$$FC = \$100$$

$$TVC = 60Q - 3Q^2 + 0.10Q^3$$

then we know,

$$TC = 100 + 60Q - 3Q^2 + 0.10Q^3$$

$$AFC = \frac{100}{Q}$$

$$AVC = 60 - 3Q + 0.10Q^2$$

Note: AVC reaches a min at

$$Q_m = -\frac{b}{2c} \text{ or}$$

$$AVC = MC$$

$$ATC = \frac{100}{Q} + 60 - 3Q + 0.10Q^2$$

$$MC = \frac{d(VC)}{dQ} = 60 - 6Q + 0.30Q^2$$

2. Cost Curves

Fig. 1

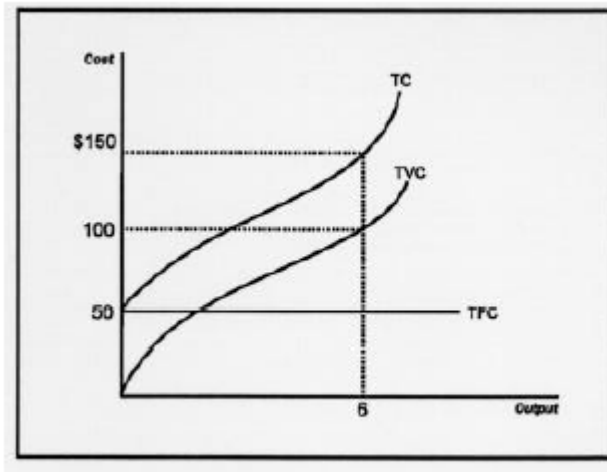
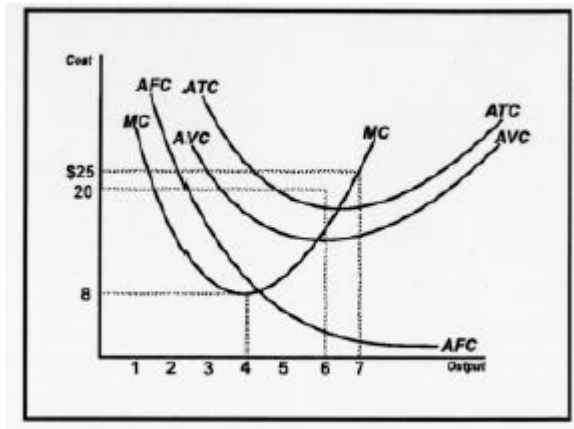


Fig. 2



The curves above show typical shapes of a firm's total cost, total variable cost and total fixed cost curves. Total fixed cost is constant at \$50 for all levels of production. Total cost and total variable cost increase with higher levels of output. Note that total fixed cost and total variable cost always add to total cost.

Relationship of TFC, TVC, and TC

- TFC is constant and unaffected by output level
- TVC is always increasing; increasing at a decreasing rate first and then at an increasing rate
- TC has same shape as TVC; but higher by the vertical distance equal to TFC

Note: These characteristics based on “typical” production function; shapes of cost curves depend on characteristics of the underlying production function.

Average and Marginal cost curves

- AFC – always declining but at a decreasing rate
- ATC and AVC – U-shaped; declining at first, reaching a minimum, and then increasing at higher output levels
- Vertical diff. bet. ATC and AVC is the AFC (changes with output level)
- ATC and AVC have minimum points at different output levels
- MC – generally increasing; with a “typical” production function, MC decreases over a short range before starting to increase
- MC hits AVC and ATC at its minimum points - as long as marginal is below average, average is decreasing (and vice-versa)

3. Short-Run Cost Functions

In addition to measuring the costs of producing a given quantity of output, economists are also concerned with determining the behavior of costs as output is varied over a range of possible values.

The behavior of costs is expressed in terms of a cost function.

4. Total Cost Function

Total Cost = sum of all costs; FC + TVC

5. Long-Run Cost Functions

The long-run cost function is obtained directly from the production function by finding the expansion path. Remember, the expansion path for a production process consists of the combinations of inputs X and Y for each level of Q that satisfy the optimality criterion:

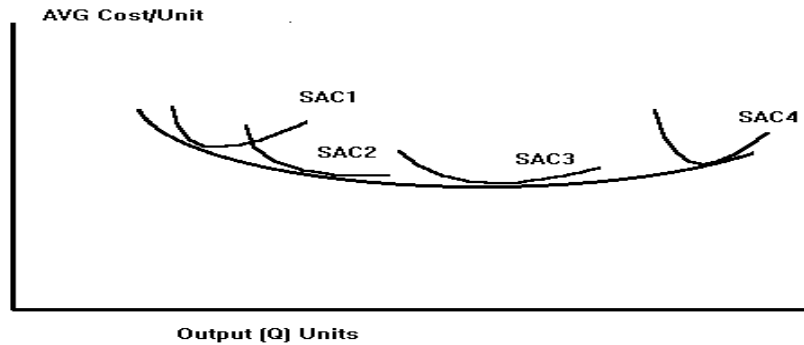
$$\frac{MP_x}{C_x} = \frac{MP_y}{C_y}$$

Over the long-run planning horizon, the firm can choose the combination of inputs that minimizes the cost of producing a desired level of output. Using the existing production methods and technology, the firm can choose:

- The plant size,
- Types and sizes of equipment,
- Labor skills, and
- Raw materials

that, when combined, yield the lowest cost of producing the desired amount of output.

The long-run average cost function (LAC) consists of the lower boundary (envelope) of all the short-run cost curves.



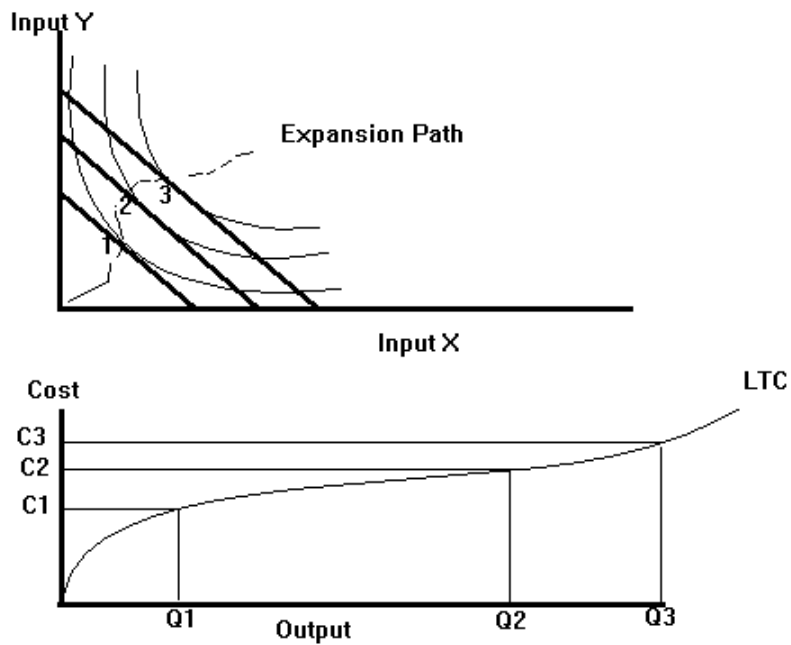
The relationship between LTC, LMC, and LAC is as follows-- The long-run cost function is obtained directly from the production function by first finding the expansion path for the given production process.

The expansion path for a production process consists of the combinations of inputs X and Y for each level of output Q that satisfy the optimality criterion developed earlier.

Recall we derived the condition that the MRTS between two inputs must be equal to the ratio of the unit costs of the two inputs for a given input combination to be an optimal solution to either the output-maximization or cost-minimization problem.

$$MRTS = \frac{C_x}{C_y}$$

Graphically, the optimal input combination occurred at the point where the production isoquant was tangent to the isocost line.

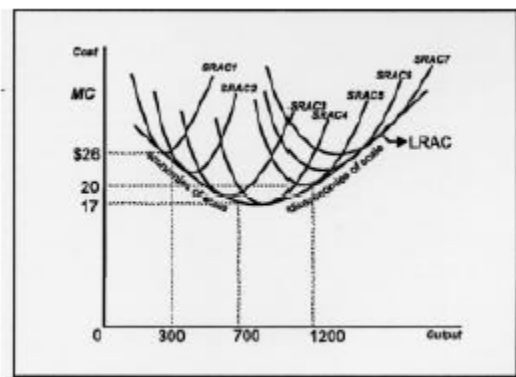


Note:

$$LAC = \frac{LTC}{Q} \text{ and } LMC = \frac{\Delta LTC}{\Delta Q}$$

6. The Long Run Average Cost Curve

The long-run average cost curve is derived from a number of short-run average cost curves. For each fixed plant size (short run), you look at the lowest costs for that size plant. These bottom portions of the different short run cost curves make up the long run average cost curve.



A firm's long run average cost curve is the "envelope" of many short run average cost curves. All inputs are variable and the firm has the choice of building or changing to a

variety of plant or facility sizes. A small operation (SRAC1), which wants to produce 300 Units, will have average costs of \$26. A larger one, which produces 700 units, can produce each product for \$17 (economies of scale). When the firm gets too large (SRAC6), average costs rise to \$20 (diseconomies of scale).

7. Increasing, Decreasing, and Constant Returns to Scale

Note that increasing returns to scale is closely associated with the concept economies of scale (the downward sloping part of the long run average total cost curve.) Decreasing returns to scale relates to diseconomies of scale (the upward part of the curve).

Increasing returns to scale occurs when a firm increases its inputs, and a more than proportionate increase in production results. For example, one year a firm employs 200 workers and 50 machines and produces 1000 products. A year later it increases the number of workers to 400 and the machines to 100 (inputs doubled) and the output rises to a level of 2500 (more than doubled).

Increasing returns to scale is often accompanied by decreasing long run average costs (economies of scale). A firm which gets bigger may experience this because of increased specialization, more efficient use of large pieces of machinery (for example, use of assembly lines), volume discounts, etc.

Decreasing returns to scale happens when the firm's output rises by less than the percentage increases in inputs. In the last example, had the firm's output risen to 1500, we would experience decreasing returns to scale.

Decreasing returns to scale can be associated with rising long run average costs (diseconomies of scale). An organization may become too big, thus creating too many layers of management, too many departments, and too much red tape. This leads to a lack of communications, inefficiency, and delays in decision-making.

Constant returns to scale occurs when the firm's output rises proportionate to the increase in inputs. What would output have to be for this to take place?

8. Marginal Utility

Marginal utility is the additional satisfaction one gets from consuming one more item of a good or service. Satisfaction is measured in utils (use your imagination). Let's say that you are about to eat a pizza consisting of 6 slices. The first piece might give you 140 utils of satisfaction (you were starving!). The second slice, your hunger somewhat satisfied after the first, yields you only 60 utils. The third's down to 20 utils and a possible fourth, if forced upon you, could produce negative additional utils (your total would drop.)

9. The Law of Diminishing Marginal Utility

Referring back to the example in the previous objective, you can see that the marginal utility declines as the person consumes more slices. This is typical of almost all (beer and other substances known to be harmful to body and soul may be sole exceptions...) consumption: the more you have of something the less the additional unit is worth to you. This phenomenon explains why if you are shopping in a supermarket you only buy a limited quantity of goods. As the marginal utility of, for example, the fifth orange declines, you may decide that this orange is not worth your additional expense.

10. Application of Cost Concepts

Profit-maximizing rules (Short-run)

If price or MR > min. ATC (or TR > TC), produce output where MR = MC

If min AVC < price or MR < min. ATC (or TVC < TR < TC), produce where MR = MC;

But will incur a loss between zero and TFC (loss minimized)

If price or MR < min ATC, do not produce and loss will equal TFC

Profit-maximizing rules (Long-run)

If price or MR > ATC, produce where MR = MC

If price or MR < ATC, stop production and sell fixed assets

11. Isoquants and the Producer's equilibrium

When producing a good or service, how do suppliers determine the quantity of factors to hire? Below, we work through an example where a representative producer answers this question.

Let's begin by making some assumptions. First, we shall assume that our producer chooses varying amounts of two factors, capital (K) and labor (L). Each factor has a price that does not vary with output. That is, the price of each unit of labor (w) and the price of each unit of capital (r) are assumed constant. We'll further assume that w = \$10 and r = \$50. We can use this information to determine the producer's total cost. We call the total cost equation an isocost line (it's similar to a budget constraint).

The producer's isocost line is:

$$10L + 50K = TC \quad (1)$$

The producer's production function is assumed to take the following form:

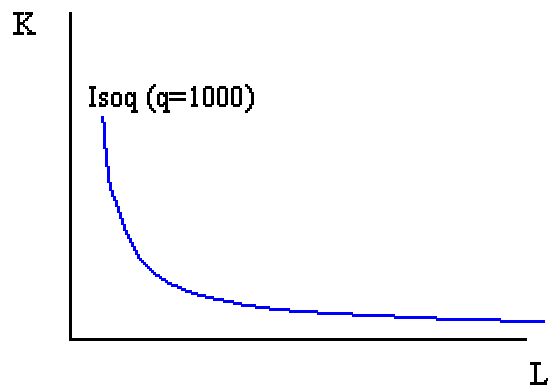
$$q = (KL)^{0.5} \quad (2)$$

Our producer's first step is to decide how much output to produce. Suppose that quantity is 1000 units of output. In order to produce those 1000 units of output, our producer must get a combination of L and K that makes (2) equal to 1000. Implicitly, this means that we must find a particular isoquant.

Set (2) equal to 1000 units of output, and solve for K. Doing so, we get the following equation for a specific isoquant (one of many possible isoquants):

$$K = 1,000,000/L \quad (2a)$$

For any given value of L, (2a) gives us a corresponding value for K. Graphing these values, with K on the vertical axis and L on the horizontal axis, we obtain the blue line on the graph below. Each point on this curve is represented as a combination of K and L that yields an output level of 1000 units. Therefore, as we move along this isoquant output is constant (much like the fact that utility is constant as we move along an indifference curve).

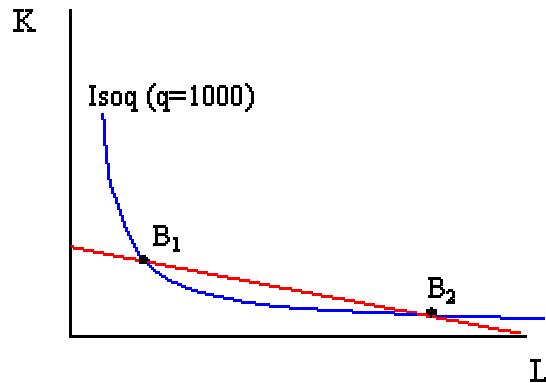


How much K and L should the producer hire? The answer is that our choice must exist somewhere on this isoquant. If each possible choice must lie on this isoquant, then our basis for choosing one "best" combination should be to choose the least cost combination. Let's experiment with some different possibilities, by plugging values for L and K into the isocost equation above. Each combination should yield output of 1000 units. Several combinations, and their specific total cost are given as follows:

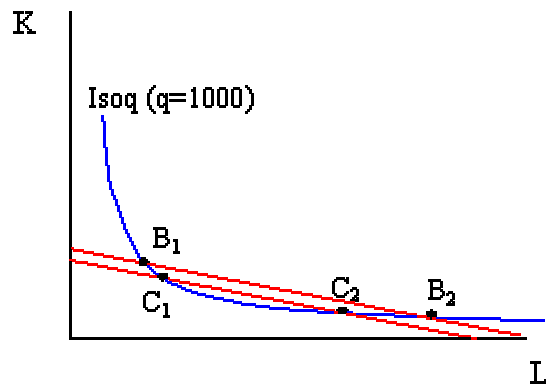
L	K	TC
1,000	1,000	60,000
5,000	200	60,000
10,000	100	105,000

100 10,000 501,000

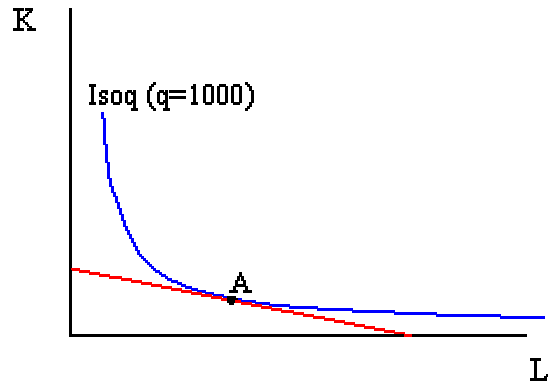
Now, the firm's goal is to produce 1000 units at the lowest possible TC. The lowest total cost on the table is \$60,000, so we can start there. There are two choices which yield this total cost. They are represented below as B_1 and B_2 .



Is there a lower cost combination of L and K available? Yes, we can produce at a total cost of \$52,500 by employing either 250 units of K and 4000 units of L, or 800 units of K and 1250 units of L. This appears on the graph below.



The second isocost, where $TC = \$52,500$, rests below the former isocost, where $TC = \$60,000$. If we continue to find lower and lower levels of total cost that provide us with 1000 units of output, then we will clearly reach lower and lower points on the isoquant. Eventually, we can find a level of total cost that involves a tangency between the isocost and isoquant. This is pictured below at point A.



In addition to being the lowest cost combination of L and K that produces 1000 units of output, pt. A involves a tangency point between the isoquant and isocost. That is, the slopes of these curves at pt. A are equal. The slope of the isocost is w/r , or $-1/5$. The slope of the isoquant is the ratio of the marginal products, MP_L/MP_K , which is given as the marginal rate of technical substitution (MRTS). Using calculus, it is possible to derive the MRTS as $-K/L$. Point A satisfies the condition that $K/L = 1/5$.

We can solve for K^* and L^* at pt. A, using (1), (2a) and the fact that, at pt. A, $K/L = 1/5$. First, substitute (2a) into (1) and the equation $K/L = 1/5$. We're left with:

$$10L + 50(1,000,000/L) = TC$$

and

$$(1,000,000/L)/L = 1/5$$

Solve the second equation for L, substitute that result into the first equation to get the lowest value for TC (TC^*).

$$TC^* = \$44,721.36$$

Once you have TC^* , you can substitute this value into the isocost equation above ($10L + 50,000,000/L = TC$) and then solve for L^* (rounded to the nearest whole number).

$$L^* = 2,236$$

Going back to (1), we can substitute in L^* and TC^* , to get K^* .

$$K^* = 447$$

12. Economies of Scale

Declining long-run average costs over the lower part of the possible outputs are usually attributed to economies of scale. Economies of scale occur over the range of the long-run

cost function which corresponds to increasing returns to scale of the production function. Where do economies of scale come from?

1. Plant Economies

- Specialization in the use of labor and capital
- Indivisible nature of many types of capital equipment
- Purchase price of different sizes of equipment

2. Firm Economies

- Materials procurement / quantity discounts
- Economies in raising funds (capital procurement)
- Sales promotion
- Technological innovation
- Management

13. Glossary of terms

1. Cost-- The sacrifice incurred whenever an exchange or transformation of resources takes place.
2. Sunk cost-- A cost incurred regardless of the alternative action chosen in a decision-making problem.
3. Cost function-- A mathematical model, schedule, or graph that shows the cost (such as total, average, or marginal cost) of producing various quantities of output.
4. Opportunity cost-- The value of a resource in its next best alternative use. Opportunity cost represents the return or compensation that must be foregone as the result of the decision to employ the resource in a given economic activity.
5. Marginal cost-- The incremental increase in total cost that results from a one-unit increase in output.
6. Cost elasticity-- A measure that indicates the percentage change in total costs associated with a 1-percent change in output.
7. Economies of Scale: Declining long-run average costs as the level of output for the firm (or production plant) is increased. The decline in costs is generally attributed to production or marketing advantages.
8. Diseconomies of scale-- Rising long-run average costs as the level of output is increased.
9. Minimum efficient scale-- The output level at which long run average costs are first minimized.
10. Capacity-- The output level at which short-run average costs are minimized.
11. Operating leverage-- The use of assets having fixed costs (e.g. depreciation) in an effort to increase expected returns.